Abstract: Cycloidal speed reducers are special high ratio transmissions developed to reduce speed. They are able to transmit and high power also. The applicability is in almost all operations of machinery, particularly where positioning accuracy is required (medical equipment, instrumentation and control, robotics, etc.). During function power loss occurs due to the friction between the contacting elements of this gear (elements of gearing, bearings, bearing on eccentric rollers on the pins) The power lost due to friction in kinematic couplings turns into heat, phenomenon that heats the reduction gears. A quantity of the heat is released into the atmosphere through the housing while another part is taken by the lubricating oil. The amount of heat produced in reducer during operation is determined considering its overall performance, respectively computing power losses of kinematic couplings. From the condition of thermal equilibrium (quantity of heat equal to the amount of heat released) the operating temperature is computed. If temperature exceeds the admitted maximum value it must be decreased by increasing the reduction gear housing’s surface and the oil amount.

Keywords: cycloidal speed reducer, friction, power, heat

1. Description of cycloidal speed reducers with bolts

Cycloidal speed reducers with bolts are special gears obtained as a particular case of cycloidal planetary gears, for which the engaging line is represented by a circle arc, and the angle of pressure is variable [1,5]. The cycloidal speed reducer with bolts is composed of two main functional parts: the actual cycloidal gear with bolts and the transversal homokinetic bolted joint.
The central element of the cycloidal planetary gears with bolts [1], (Fig. 1) is the input shaft 1. It holds the eccentric plate 3. On this are mounted – through cylindrical roller bearings 10 – are one, two or more satellite wheels endowed with exterior cycloidal profiled teeth. Through the motion the eccentric plate, due to its eccentricity engages the teeth of the satellites with the central wheel 4 whose inner teeth consist of five circular disposed equidistant embedded bolts.

The transversal homokinetic joint consists of the output shaft 7 (coaxial with the input shaft 1) ending in a flange. The flange sustains the circularly disposed bolts 8 which will take over the motion and the torque from the satellite gears. It interacts through a number of dents equal to the number of pins 8, inscribed in a circle, to the satellites 2 and 2’.

In order to reduce friction between the satellite wheels 2 and 2’ and bolts 5 of the central wheel 4, also between the satellite wheels 2 and 2’ and bolts 8 of the homokinetic joint, the bolts are fitted with rollers 6 and 9, which rotate freely about its bolt’s axis.

2. The operational mode of cycloidal gear with bolts

The $M_m$ torque is transmitted from the input shaft 1 to the eccentric 3 and the roller wheels 10 which interact with satellite gears 2 and 2’, causing their nearing or distancing from the bolts 5 of the fixed crown 4. Reaction forces created in the rotation couplings formed between the bearings and the satellites tend to rotate the satellites in the direction of rotation of the input shaft 1. The nearing of satellites towards the central wheel causes their interacting with bolts 5 of the fixed wheel 4.
In the superior coupling, reactions $R_5$ and $R_5'$ are created at the application point, tangent to the bolts (bushings 6) with the cycloidal profile of the satellite wheels. They are oriented alongside the common normal of the two tangent profiles in their contact point. Those reactions cause the satellites to rotate in reverse sense compared to the rotation of the input shaft with speed $n = n_m/z_4$, where $n_m$ is the input shaft speed, and $z_4$ is the number of bolts 5 of fixed wheel 4.

Collection of the motion and torque from the satellite wheels is achieved with the help of the homokinetic joint, and transmitted to output shaft 7.

3. Thermal calculations

In the upper couplings with linear contact (formed by the interaction of bolts 5 with rollers 6, by rollers 6 with the cycloidal profile of satellite gears 2 and 2', by bolts 8 of the homokinetic joint with rollers 9 and of those with the circular profile of pits on satellites 2 and 2'), also through the interaction between the rollers of bearing 10 of the eccentric 3 with satellite gears 2 and 2', friction appear. As a result power losses and the transmission’s efficiency decreases. The amount of heat $Q$ released during operation is calculated using the formula (1)

$$Q = 260(1 - \eta)N \text{ [kcal/h]}$$

where

- $N$ – the driving shaft power in [kW];
- $\eta$ – the global efficiency of reducer.

The amount of heat released is influenced by the efficiency of the gearbox, whose value is calculated with the equation (2):

$$\eta = \eta_1 \eta_2 \eta_3 \text{ [where]}$$

$\eta_1$, $\eta_2$, and $\eta_3$
- \(\eta\) – the efficiency of the gearing;
- \(\eta_u\) – efficiency of the oil, taking into account its bubbling losses;
- \(\eta_l\) – efficiency of bearings.

The oil efficiency \(\eta_u\) is calculated by the relation (3)

\[
\eta_u = \frac{1,36N - 0,082V B \sqrt{\frac{V \nu - 200}{z_2 + z_4}}}{1,36N}
\]

where

- \(N\) – the transmitted power [kW];
- \(B\) – the total width of the satellites [m];
- \(V\) – the peripheral speed of satellites [m/s];
- \(\nu\) – the viscosity of the oil [cst];
- \(z_2, z_4\) – number of teeth of satellite gears, and the number of bolts.

The transmission efficiency \(\eta_t\) can be calculated according to the specific loss coefficient \(\psi\) representing power loss if the gearing is considered an ordinary gear. The relation to calculate the efficiency in this case is [2]:

\[
\eta_t = \frac{1 - \psi}{1 + z_2 \psi}
\]

(4)

The loss coefficient \(\psi\) has three components according to relation (5):

\[
\psi = \psi_{26} + \psi_{29} + \psi_{32}
\]

where

- \(\psi_{26}\) – the specific loss of engagement of fixed crown gear bolts with the satellite gear;
- \(\psi_{29}\) – the specific loss of the coupling of the homokinetic joint;
- \(\psi_{32}\) – the specific loss of the coupling between the eccentric and the satellite.

To determine the loss coefficient \(\psi\), three coefficients will be calculated as follows:

a) Specific loss coefficient of cycloidal gearing with bolts:

\[
\psi_{26} = K_a \mu_a / z_4
\]

where

- \(K_a\) – a coefficient that is chosen in dependence with the correction coefficient \(\zeta\) according to (Fig. 3);
- \(\mu_a\) – the gearing friction coefficient.

In [2] we recommend for \(\mu_a\) the highest value of either the coefficient of friction between bolts 5 and rollers 6, \(\mu_{56}\), or \(\mu_{26}\) the friction coefficient between the teeth and roller 6 (Fig. 4) [3]. Experimentally it was found that power losses
are to a greater extent influenced by friction between the bolts and rollers (due to different conditions of lubrication) than friction between rollers and the cycloidal profile. The friction coefficient $\mu_{26}$ depends on the rolling speed, and is determined according to figure 5.

**Figure 3:** Value of the coefficient $K_a$, depending of $\zeta$

**Figure 4:** The interaction cycloidal-wheel of profile, wheel-bolt

**Figure 5:** Value of the coefficient $\mu_{26}$, depending of $v_r$
b) Specific loss coefficient in the transverse coupling with bolts shall be determined by the relation (7)

\[ \psi_{28} = \frac{4e \mu_e}{\pi R_v} \]  

- \( e \) – the eccentricity of the gear;
- \( \mu_e \) – the equivalent friction coefficient,
- \( R_v \) – the radius of placement of bolts on the transverse coupling.

Identical with those considered under point a), friction occurs between the profile pits of radius \( r_o \) on satellite gears 2 and 2' and rollers 9 of radius \( r_o \), and between rollers 9 and bolts 8 (Fig. 6). According to [2] the coefficient of friction \( \mu_e \) is chosen according to (Fig. 6) in dependence with the speed of rolling \( \nu_r \), which is calculated with the equation (8):

\[ V_r = (r_v + r_{o0})(\omega_2 + \omega_3) \text{[m/s]}, \]  

- \( \omega_2 \) – the angular velocity of the satellite wheel [m/s];
- \( \omega_3 \) – the angular velocity of the eccentric [m/s]
- \( r_v, r_{o0} \) – the radius of the pit, respectively the roller 9 [m].

c) Specific loss coefficient in the bearing of the eccentric: according to [2] it is given by:

\[ \psi_{32} = 1.63 \left( 1 + \frac{D_3}{d_r} \right) \frac{f_r}{r_v} \sqrt{1 + \left( \frac{4 \ r_v}{\pi R_v} - K_y \right)^2}, \]  

- \( D_3 \) – the diameter of eccentric 3;
- \( d_r \) – the diameter of rollers in bearing 7;
- \( f_r \) – coefficient of rolling friction;
- \( r_v \) – radius of the base cycle used in the satellite profile generation;
- \( R_v \) – radius of placement of bolts on the homokinetic joint;
- \( K_y \) – coefficient of the correction applied to the gearing (Fig. 7).

Considering the three components of the loss coefficient, formula (5) becomes:

\[ \psi = \frac{K_y \mu_e}{\varepsilon_4} + \frac{4e \mu_e}{\pi R_v} + 1.63 \left( 1 + \frac{D_3}{d_r} \right) \frac{f_r}{r_v} \sqrt{1 + \left( \frac{4 \ r_v}{\pi R_v} - K_y \right)^2} \]

Using formula (10) formula (4) can be used to determine the global efficiency of the transmission. The efficiency \( \eta \) of bearing elements depends on the type of bearing and the peculiarities of the construction of the gearbox.
By example the analyzed type of gearbox uses two types of bearings. Considering that a bearing of high performance ($\eta_{per} = 0.99$), [4] is used it results $\eta_b = 0.98$. Computing $\eta_b$ using equation (3), $\eta_p$ from equations (4) and (10) and considering the previous determined lubrication efficiency coefficient the global efficiency of the transmission becomes possible to be calculated by equation (2). After this, the quantity of heat Q caused by power losses in the speed reducer will be computed using relation (1).

The quantity of heat that can be released in the environment $Q_1$ by the speed reducer casing is given by [8]:

$$Q_1 = \alpha S (t - t_0) \text{[kcal/h]},$$  \hspace{1cm} (11)

- $\alpha$ – the coefficient of heat transmission through the surface $S$, $\alpha = 10...18$ [kcal/m$^2$°C];
- $S$ – the surface of the housing [m$^2$];
- $t$ – the limit temperature of the transmission [°C];
- $t_0$ – the ambient temperature [°C], ($t_0 = 20$ °C).
The operating temperature of the speed reducer is determined from the condition of thermal equilibrium, i.e. heat $Q$ produced during the operation of the reducer is equal to the amount of heat exhausted to the environment $Q_1$:

$$ Q = Q_1. \quad (12) $$

In the case of natural cooling and a long-term operation, operating temperature limit is given by the formula:

$$ t = t_0 + \frac{860(1-\eta)}{\alpha S}. \quad (13) $$

For a good operation of the reducer we impose that the normal operating temperature should not exceed the admissible value $t_a = 70...85 \, ^\circ\text{C}$ [4]. Increasing the operating temperature above the permissible value causes a large decrease in viscosity of the oil which leads to improper lubrication and therefore the premature destruction of the reducer components [7].

If operating temperatures calculated according to the equation (13) result greater that the admissible limit value, then a fan can be mounted on the input shaft to help cool off portions of some of the reducer housing.

Another solution is to increase the surface $S$ by manufacturing a ribbed outer enclosure.

### 3. Conclusion

When designing bolted cycloidal speed reducers the condition of thermal equilibrium must be taken into consideration.

If the calculated operating temperature results higher than the permissible temperature, when designing the speed reducer housing, a ribbed model can be selected in order to increase its outer surface, acting as a heat sink.

### References