Two-Stage Kalman Filtering for Indoor Localization of Omnidirectional Robots

Lőrinc MÁRTON, Katalin GYÖRGY

Department of Electrical Engineering,
Faculty of Technical and Human Sciences,
Sapientia Hungarian University of Transylvania, Tg. Mureș,
e-mail: martonl@ms.sapientia.ro

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Abstract: In this study a sensor fusion technique was developed for indoor localization of omnidirectional mobile robots. The proposed sensor fusion method combines the measurements made by an indoor localization system (e.g. ultrasound based localization) with the measurements that comes from an IMU (Inertial Measurement Unit). It was taken into consideration that the measurements made by the ultrasonic sensor are available with lower measurement rate than the IMU measurements. To tackle this problem a two-stage Kalman filter was developed. The first stage estimates the orientation of the robot at the measurement rate of the IMU using a linear Kalman filter. The second stage estimates the position and velocity of the robot. The prediction of the robot’s position is performed based on the robot model at the update rate of the IMU. The correction phase is performed at the update rate of the external localization system using nonlinear Kalman filtering techniques. All the measurements were considered noisy and in the case of the IMU the measurement biases were also taken into consideration. For the implementation of the nonlinear Kalman filter, a discretized nonlinear omnidirectional robot model was developed. It was necessary as the classic unicycle model cannot be directly applied in the case of the omnidirectional robot. Simulation measurements were performed to evaluate the performances of the proposed two-stage filter. For the implementation of the second stage, two approaches were tested namely the Extended Kalman Filter and the Unscented Kalman Filter.

Keywords: Robot sensing systems, Motion estimation, Kalman filtering, Sensor Fusion.

1. Introduction

The sensor fusion is an effective method to solve the precise localization problem of mobile robots. According to this technique more than one sensor are applied to obtain the position of the robot and to combine effectively the
different measurements to generate the estimated system states. The sensor fusion allows the mitigation of the limitations of different sensor, obtaining a more accurate position of the robot.

The Kalman Filtering technique is an efficient way for sensor fusion in robotic applications [1]. When the measurement model of the robot is linear the Linear Kalman Filter is suitable to solve the sensor fusion problem. In the case of nonlinear robot models Extended Kalman Filtering techniques of the Unscented Kalman Filtering can be applied. An alternative to the Kalman Filter can be the Particle Filter based robot state estimation [2].

There are several ways to solve the indoor localization of the robot; the most common is the vision based localization [3]. Another promising approach is based on ultrasonic signal receiver - emitter pairs. The emitters or receivers are placed over the robot’s workspace and its pair is placed on the robot. These systems measure the Time of Flight of the signals traveling between the synchronized receivers and transmitters. By assuming that the spatial positions of the receivers are known, multiple measurements can be combined with trilateration to find the location of the robot [4].

The orientation of a mobile robot related to a given axis can be determined using dedicated sensors such as magnetometers [5].

If precise time synchronization and effective signal detection is provided, a well calibrated ultrasound based localization system can provide sub-centimeter position accuracy. However the standard Time of Flight ultrasonic range finders are only able to provide tens of readings per second. This update rate is not enough for fast robot control applications. To handle this problem, the Time of Flight measurement based position computation can be fused with inertial measurements to provide much faster position estimation. These sensor fusion methods use the model of the robot and measurements from accelerometers and gyroscope sensors [6].

The goal of the current work is to develop a sensor fusion technique for omnidirectional mobile robots which combines the measurements of a robot localization system having low measurement update rate with the measurements of the Inertial Measurement Unit (magnetometer, accelerometer, gyroscope). The two-stage Kalman filter introduced in this work is similar to the quadrotor helicopter localization method presented in [7].
2. Robot Modeling for Sensor Fusion

2.1. Unicycle robots

Consider a unicycle-type robot that moves in a horizontal plane. The coordinates of the robot’s position \((x, y)\) are given in an inertial reference frame \((x_Wy_W)\), see Fig. 1. Another frame is attached to the robot’s body \((x_Ry_R)\) which origin is in the center of gravity of the robot. The orientation of the robot in the reference frame is defined as the angle between the \(x\) axes of the reference frame and robot’s frame \((\theta)\).

\[
\begin{align*}
\dot{x} &= \cos(\theta) \cdot v, \\
\dot{y} &= \sin(\theta) \cdot v, \\
\dot{\theta} &= \omega, \\
\dot{v} &= a.
\end{align*}
\]

Figure 1: Unicycle robot.

Due to the kinematic constraint in the unicycle-type robot’s motion, the robot cannot generate velocity along the \(0y_R\) axis. The direction of the robot’s velocity vector always corresponds to the \(x\) axis of the robot’s frame. The motion of the mobile robot in a two dimensional reference frame is described by:

Here \(v\) is the velocity of the mobile robot, \(\omega\) denotes the angular velocity of it. The notations \(x\) and \(y\) stand for the robot’s coordinates in the reference frame and the angle \(\theta\) denotes the robot’s orientation. The acceleration of the robot along the \(0x_R\) axis \((a)\) is the first order derivative of \(v\).
2.2. Omnidirectional robots

In the case of the omnidirectional robot model it has to be considered that this type of robot can generate angular velocity and linear velocity independently. This fact makes the control design of the omnidirectional robot simpler. However, during the model development for sensor fusion the inter-influence of the independent linear and rotational motions has to be taken into consideration.

Consider an omnidirectional robot that moves in a horizontal plane. The coordinates of the robot’s position \((x, y)\) are given in an inertial reference frame \((x,w0,yw)\), see Fig. 2. Another frame is attached to the body \((x_R,y_R)\) of the robot. The orientation of the robot in the reference frame is defined as the angle between the \(x\) axes of the reference frame and robot’s frame \((\alpha)\).

The robot can generate velocity along both \(0x_R\) and \(0y_R\) axes, i.e. the direction of the robot’s velocity vector does not necessarily correspond to the \(x\) axis of the robot’s frame. The motion of the mobile robot in a two dimensional reference frame is described by:

\[
\begin{align*}
\dot{x} &= \cos(\theta) \cdot v, \\
\dot{y} &= \sin(\theta) \cdot v, \\
\dot{\alpha} &= \omega.
\end{align*}
\]  

**Figure 2**: Omnidirectional robot.

Here \(\theta\) is the angle between the velocity vector and the \(x\) axis of the reference frame.

In the robot’s frame the components of the velocity vector are denoted by \(v_x\) and \(v_y\). The relation between \(\theta\) and \(\alpha\) is:
The kinematic relationship between acceleration measurements and velocity components can generally be formulated as [9]:

\[
\begin{align*}
\dot{v}_x &= a_x + \omega \cdot v_y \\
\dot{v}_y &= a_y - \omega \cdot v_x
\end{align*}
\]

(4)

Consider an Inertial Measurement Unit (IMU) mounted in the robot’s center which contains accelerometer, gyroscope and a magnetometer. The magnetometer is applied for orientation measurement. Using the IMU acceleration of the robot along the \(x\) and \(y\) axes (\(a_x, a_y\)), the angular velocity around the axis perpendicular to the robot’s frame (\(\omega\)) and the orientation of the robot (\(\alpha\)) is measured. The IMU measurements are corrupted by noise:

\[
\begin{align*}
\hat{a}_x &= a_x + b_{ax} + w_{ax}, \\
\hat{a}_y &= a_y + b_{ay} + w_{ay}, \\
\hat{\omega} &= \omega + b_{\omega} + w_{\omega}, \\
\hat{\alpha} &= \alpha + w_{\alpha},
\end{align*}
\]

(5)

where \(b_{ax}, b_{ay}\) and \(b_{\omega}\) are constant unknown biases and the stochastic noise components \(w_{ax}, w_{ay}, w_{\omega}\) and \(w_{\alpha}\) are assumed zero mean with normal distribution: \(w_{ax}, w_{ay} \sim N(0, \sigma_a), w_{\omega} \sim N(0, \sigma_\omega), w_{\alpha} \sim N(0, \sigma_\alpha)\).

The coordinates \(x\) and \(y\) are measured by a localization system and the measurements are also considered to be affected by zero mean value stochastic noise with normal distribution \((N(0, \sigma))\).

2.3. Discretized Two-Stage Model for Sensor Fusion

Consider that the IMU provides the measurements with \(T\) measurement period and the external localization system has a measurement period \(kT\), where \(k\) is a strictly positive integer. To deal with multiple measurement rates a two-stage discrete time modeling is proposed. For the numerical approximation of
the continuous time model, the Euler integration method can be applied, i.e. 
\( \dot{\alpha} \approx (\alpha[k] - \alpha[k-1])/T. \)

**Stage 1.** The angular position and the bias of the velocity offset are estimated:

\[
\begin{align*}
\alpha[k] &= \alpha[k-1] + T \cdot (\alpha[k-1] + b_\alpha[k-1]) \\
\hat{b}_\alpha[k] &= b_\alpha[k-1]
\end{align*}
\]

As measured output, the orientation of the robot is considered; the input is the angular velocity.

**Stage 2.** Based on the estimated orientation in Stage 1, from the model and measurement equations the position yields the model for robot pose estimation

\[
\begin{align*}
x[k] &= x[k-1] + T \cdot \cos \left( \alpha[k-1] + \arctan \left( \frac{v_y[k-1]}{v_x[k-1]} \right) \right) \cdot \sqrt{v_x[k-1]^2 + v_y[k-1]^2}, \\
y[k] &= y[k-1] + T \cdot \sin \left( \alpha[k-1] + \arctan \left( \frac{v_y[k-1]}{v_x[k-1]} \right) \right) \cdot \sqrt{v_x[k-1]^2 + v_y[k-1]^2}, \\
v_x[k] &= v_x[k-1] + T \cdot \alpha[k-1] + b_{ax}[k-1] - \alpha[k-1] \cdot v_x[k-1], \\
v_y[k] &= v_y[k-1] + T \cdot \alpha[k-1] + b_{ay}[k-1] - \alpha[k-1] \cdot v_y[k-1], \\
\hat{b}_{ax}[k] &= b_{ax}[k-1], \\
\hat{b}_{ay}[k] &= b_{ay}[k-1].
\end{align*}
\]

The measured outputs are the coordinates \( x[k], y[k] \) the inputs are the acceleration components of the robot \( (a_x[k], a_y[k]) \) and the orientation and angular velocity measurements \( (\omega[k], \alpha[k]) \). As the position measurement update rate is lower than the IMU measurement update rate, between two position measurements the model above is used for prediction. When a new position measurement is available, the update phase of the estimation is executed.

In the case of the localization task the states, outputs and inputs are defined as:

\[
\begin{align*}
X &= \begin{pmatrix} x & y & v_x & v_y & b_x & b_y \end{pmatrix}^T \\
Y &= \begin{pmatrix} x & y \end{pmatrix}^T \\
U &= \begin{pmatrix} a_x & a_y \end{pmatrix}^T
\end{align*}
\]

Note that the proposed modeling technique presented here for omnidirectional robots can also be applied to the unicycle robots introduced in subsection 2.1.
3. A short review of the Linear, Extended and Unscented Kalman Filter

The general framework for nonlinear state estimation methods is based on a nonlinear discrete time state space model

\[
\begin{align*}
X[k] &= f(X[k-1], U[k-1]) + w[k-1] \\
Y[k] &= h(X[k], U[k]) + v[k]
\end{align*}
\]  

(9)

where \(X[k]\) is the \(n\)-dimensional state vector, \(U[k]\) is the \(m\)-dimensional input vector and \(Y[k]\) is the noisy output vector (\(p\)-dimensional) of the system. The \(f\) and \(h\) are nonlinear continuous functions. The \(w[k]\) is an \(n\) dimensional process noise sequence and \(v[k]\) is a \(p\) dimensional observation (measurement) noise sequence. Both noises (vectors) are Gaussian (normal distribution), independent random processes with zero means and known time invariant covariance matrices:

\[
\begin{align*}
w &\sim N(0, \sigma_w) \quad \text{cov}\{w\} = Q \quad E\{w\} = 0 \\
v &\sim N(0, \sigma_v) \quad \text{cov}\{v\} = Q \quad E\{v\} = 0
\end{align*}
\]  

(10)

The objective of the estimation process is to calculate the \(X[k]\) recursively from the output measurements \(Y[k]\). In accordance with Bayesian theory this means to determine recursively the estimation of \(X[k]\) if the following data is given: \(Y[1], \ldots, Y[k]\) (up to time \(k\)). We consider that the initial probability distribution function (pdf) of the state vector \(pdf(X[0]|Y[0])\) is known and the \(pdf(X[k]|Y[1:k])\) is obtained recursively in two steps: prediction step and update (correction) step.

The linear Kalman filter (KF) can be used just for linear dynamic systems and this method propagates the mean and covariance of the pdf of the model states in an optimal way.

The Extended Kalman filter (EKF) in many ways is similar with simple Kalman filter. In this case the state distribution is propagated analytically through a linear approximation of the system around the working point. The linear approximation of the nonlinearities may introduce errors in the estimated states. The state matrix of the linear approximation is computable as:

\[
F(X[k], U[k]) = \frac{\partial f(X[k], U[k])}{\partial X} 
\]  

(11)

The Unscented Kalman Filter (UKF) is a derivative free method, and it resolves the estimation problem using a deterministic sampling approach [8]. The state distribution is also represented by Gaussian random variables, but this algorithm is using a minimal set of sample points, called sigma points. The sigma points are calculated in each step and they completely capture the true
mean and covariance of the states and are propagated through the nonlinearity. For calculating the statistics of a random variable which undergoes a nonlinear transformation we can use the unscented transformation.

4. Simulation measurements

4.1. Simulation environment

The proposed sensor fusion algorithm was tested in a simulation environment that was developed in Matlab environment. The Matlab implementation of the omnidirectional robot model has the form:

```matlab
T = 0.01; % sec
time = 1:T:20;
% robot acceleration
ps = 0.5;
a.xr = -cos(ps*time)*ps^2;
a.yr = -sin(ps*time)*ps^2;
% robot orientation
omega_r = -0.1572;
alpha_r = alpha_r + T*omega_r;
% robot motion
v.xr = v.xr + T*(a.xr + omega_r*v.yr);
v.yr = v.yr + T*(a.yr - omega_r*v.xr);
x.r = x.r + T*cos(alpha_r + atan2(v.yr, v.xr)) * sqrt(v.xr^2 + v.yr^2);
y.r = y.r + T*sin(alpha_r + atan2(v.yr, v.xr)) * sqrt(v.xr^2 + v.yr^2);
```

The measurement update period of the IMU was chosen $T = 10\text{ms}$. The inputs of the robot were the acceleration signals, which were generated as sinusoidal both along the $x$ and $y$ axis. As the two accelerations are shifted with a phase $\pi/2$ the robot motion is almost circular. A constant angular velocity was assumed during the robot’s motion.

The measurement noises were assumed in the form:
As it can be seen, both on the acceleration and on the angular velocity measurements high frequency noises and measurement biases were assumed.

The position measurements that come from the ultrasonic measurement system were taken with an update rate which is ten times smaller than the IMU's update rate, i.e. 100ms. The $x$, $y$ position measurements were also assumed to be affected by high frequency noises.

Firstly, the angular position and the bias of the angular velocity were estimated, these were taken as system states in the first stage of the sensor fusion: $(\alpha, b_\omega)$. For the implementation of this first stage a linear Kalman filter was applied in which the $Q$, $R$ and $P_{kk}[0]$ matrices were chosen as:

According to the equation (6) the matrices for the linear system to be estimated can be written as:
For the implementation of stage 2 in the prediction phase of the Extended Kalman Filter the model (7) was directly applied. The states of the estimator for the second stage are \((x, y, v_x, v_y, b_x, b_y, \alpha, \omega)\), the measured outputs are \((x, y)\), the inputs are \((a_x, a_y)\). The angular position \((\alpha)\) estimated in stage 1 and the angular velocity \((\omega)\) of the robot were also considered known parameters for stage 2. In the update phase the nonlinear system was linearized around the current estimated state. The linearized state matrix has been implemented as (see equation (11)):

\[
\begin{align*}
F_2(1,:) &= [1 \ 0 \ T \cdot \text{d}_f \cdot \text{x} \cdot \text{per} \cdot \text{d}_v \cdot \text{v} \ x \ T \cdot \text{d}_f \cdot \text{x} \cdot \text{per} \cdot \text{d}_v \cdot \text{y} \ 0 \ 0]; \\
F_2(2,:) &= [0 \ 1 \ T \cdot \text{d}_f \cdot \text{y} \cdot \text{per} \cdot \text{d}_v \cdot \text{x} \ T \cdot \text{d}_f \cdot \text{y} \cdot \text{per} \cdot \text{d}_v \cdot \text{y} \ 0 \ 0]; \\
F_2(3,:) &= [0 \ 0 \ 1 \ T \cdot \text{omega} \ T \ 0]; \\
F_2(4,:) &= [0 \ 0 \ -T \cdot \text{omega} \ 1 \ 0 \ T]; \\
F_2(5,:) &= [0 \ 0 \ 0 \ 0 \ 1 \ 0]; \\
F_2(6,:) &= [0 \ 0 \ 0 \ 0 \ 0 \ 1];
\end{align*}
\]

The partial derivatives in the first two lines of the system matrix \(F_2\) were obtained using symbolic derivation for the first two lines of the model (7):

```matlab
syms v_x v_y alpha
f_x = T*sqrt(v_x^2 + v_y^2)*sin(alpha + atan(v_y/v_x));
f_y = T*sqrt(v_x^2 + v_y^2)*cos(alpha + atan(v_y/v_x));
d.f_x.per.d_v.x = diff(f_x, v_x);
d.f_x.per.d_v.y = diff(f_x, v_y);
d.f.y.per.d.v.x = diff(f_y, v_x);
d.f.y.per.d.v.y = diff(f_y, v_y);
```

For the implementation of this second stage the \(Q, R\) and \(P_{kk}[0]\) matrices were chosen as:

```matlab
% covariance matrix of the state noises
f1.a = .5;
Q_2 = f1.a*eye(6);
% covariance matrix of the output noises
f1.p = 0.025;
R_2 = f1.p^2*eye(2);
% covariance of the initial estimation error
L = 1;
Pkk_2 = L*eye(6);
```

These values were chosen both in the case of the Extended Kalman Filter and the Unscented Kalman Filter.
4.2. Simulation results

For sensor fusion the simulation ran for 2000 IMU measurement periods, i.e. 20s.

*Fig. 3* shows the behavior of stage 1 of the estimation. As the figure shows, the Linear Kalman Filter can offer a precise estimate of the angular position and angular velocity bias in spite of noisy measurements.

![Figure 3: Stage 1: Inputs, measured outputs, estimated states.](image)

*Fig. 4* shows the estimated states generated by the Extended Kalman filter. These measurements also show that the EKF can cope with measurement noises of the accelerometer. Between the measurements the model based prediction can offer a good estimate of the robot’s motion for the time intervals between the measurement update.

The resulting motion of the robot is presented in *Fig. 5*. During the localization measurements the robot moved on an almost circular trajectory, which assures that all the states of the system change in time.

The performances of the Extended Kalman Filter were compared with the performances of the Unscented Kalman Filter applying the same Q and R matrices during the simulation. The *Fig. 6 and 7* show that the UKF approach has longer transients in the estimation errors. On the other hand, when the estimation transients lapse, the average of the estimation errors are smaller in the case of the UKF, see *Fig. 8*. The measurements shown in this figure take into account the last 1000 measurement iterations to get rid of the estimation transients.
The beneficial effects of the UKF can also be seen in Fig. 9 and 10. The UKF has a much pronounced filtering capability as the EKF and it mitigates the effect of the measurements noises more efficiently.

Figure 4: Stage 2: Inputs, measured outputs, estimated states.

Figure 5: Robot’s motion in plane during the simulation experiments.
Figure 6: Comparison of UKF and EKF: estimated states.
Figure 7: Comparison of UKF and EKF: estimation errors.
Figure 8: Comparison of UKF and EKF after the transients: sum of the normalized absolute values of estimation errors.
5. Conclusions

In this work a sensor fusion method was introduced to improve the performances of indoor mobile robot localization systems (such as ultrasonic, radio signal based). To handle the slow measurement update rate of the localization system the position measurements are extended with a position predictor that applies the robot sensor model which input is given by IMU measurements. The obtained solution is a combination of a linear and an Extended Kalman filter, a two-stage Kalman filter. The Linear Kalman Filter
works with the update rate of the IMU and it provides the orientation of the robot. The second stage is an Extended Kalman Filter provides the planar position and the linear velocity components of the omnidirectional robot. The effectiveness of the method is shown through simulation examples. The simulation experiments show that applying Unscented Kalman Filter instead of Extended Kalman Filter could lead to more accurate position estimation.

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